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# WAVELET TRANSFORM AND WAVELET APPLICATION

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## ABSTRACT

The structure of wavelets can be well understood using multi-resolution analysis (MRA). In wavelet analysis, an issue is often transformed into its wavelet domain using the proper basis. To get the desired outcomes, the issue is first addressed in the wavelet domain and then transformed back. The capability of wavelet analysis to show information in a hierarchical fashion is a crucial characteristic. Wavelets are basis functions that can simultaneously describe a signal in the time domain and the frequency domain. Similar to Fourier transforms, they can be used to approximate an underlying trace or signal. Wavelets have the advantage of being well localized in frequency and time, which enables them to handle a larger variety of signals than Fourier analysis. Wavelets analysis explains how a given function varies from one time period to the next, whereas Fourier transform analyzes the structure of a given function in terms of sinusoidal waves of varying frequencies and amplitudes. The ability to select a particular wavelet to match the type of function being evaluated makes wavelet analysis more flexible. Integrals and derivatives are fundamental calculus techniques with a wide range of uses in science and engineering. Numerous scholars are working to create various numerical methods for figuring out answers to various problems involving integrals and derivatives. Some integrals and derivatives are difficult to find precisely, while others call on special functions that are difficult to compute even on their own. Differential, integral, and integro-differential equations govern many systems in science and engineering. Because differentials and derivatives are the most reliable tools for using when describing changes numerically, differential equations appear in a variety of contexts throughout mathematics and science (related, though not quite the same). Numerous mathematical models of physical, chemical, and biological events are built using partial differential equations (PDEs), and more recently, their application has expanded to include image processing, economic forecasting, and other domains. Analytical solutions to the great majority of PDE models are impossible. As a result, it is frequently important to numerically approximation their solutions in order to investigate the predictions of PDE models of such occurrences. A rough

solution is typically expressed by functional values at specific discrete places (grid points or mesh points). An effective tool for modeling a wide range of phenomena and processes is the integral and integro-differential equation.

**KEY WORDS:** ***Wavelet, Transform, Small Wave, Fourier transform.***

## INTRODUCTION

## WAVELET TRANSFORM

Wavelet mean „small wave", so wavelet analysis is about analyzing signal with small duration finite energy functions. They transform the signal under investigation into another representation which converts the signal in a more useful form. This transformation of the signal is called wavelet transform. Wavelet transform have advantages over traditional Fourier transform for representing functions that have discontinuities and sharp peaks, and for accurately deconstructing and reconstructing finite, non-periodic and non-stationary signals. Unlike Fourier transform, we have a different type of wavelets that are used in different fields. Choice of a particular wavelet depends on thetype of application in hand. We manipulate wavelet in two ways. The first one is translation (change of position). We change the central position of the wavelet along the time axis. The second one is scaling. The wavelet transform is basically quantifies the local matching of the wavelet with the signal. If the wavelet matches with the signal well at a specific scale and location, then a large transform value is obtained. The transform value is then plotted in two- dimensional transform plane. The transform computed at various locations of the signal and for various scale of the wavelet. If the process is done in a smooth and continuous fashion, then transform is called continuous wavelet transform. If the scale and position are changed in discrete steps, the translation is called discrete wavelet transform. Note that in case of Fourier transform, spectrum is one-dimensional array of values whereas in wavelet transform,we get a two dimensional array of values. can also be solved by converting it to an equivalent Volterra integral equation. Recall that in integro-differential equations, the initial conditions are usually prescribed in the above. The study will be focused on the Volterra integro-differential equations where the kernel is a difference kernel. Having converted the integro-differential equation to an equivalent integralequation, the latter can be solved by using wavelet methods. It is obvious that the Volterra integro-differential equation involves derivatives at the left side, and integral at the right side. To perform the conversion process, we need to integrate both sides $n$ times to convert it to a standard Volterra integral equation. It is therefore useful to summarize some offormulas to support the conversion process. We point out that the first set of formulas is usually studied in calculus. However, the second set, where reducing multiple integrals to a single integral is presented.

$$\int_0^t u'(s)\,ds = u(t) - u(0),$$

$$\int_0^t \int_0^{t_1} u''(s)\,ds\,dt_1 = u(t) - tu'(0) - u(0)$$

$$\int_0^t \int_0^{t_1} \int_0^{t_2} u'''(s)ds\,dt_2\,dt_1 = u(t) - \frac{1}{2!}t^2u''(0) - tu'(0) - u(0),$$

## WAVELET ANALYSIS-FUNDAMENTALS

Daubechies gave a vast knowledge of mathematical aspects of wavelets for a proper grasp of wavelet analysis. Chui offered a clearer and more understandable introduction to wavelets in 1992, and Nguyen and Strang followed up in 1996. The mathematical function L2(R) gives the basic description of wavelets: j,k(x) = 2j/2(2jx k), where j, k Z is an orthonormal basis of the Hilbert space of finite energy functionL2(R). When operators like translation and dilation are performed on a single function, this function space is obtained. This is an abstract operation, and to explain it to a layperson, the shape of the function is not changed; rather, it is merely transferred to a position/time of interest after squeezing or spreading it.

## WAVELET TRANSFORMS

The continuous wavelet transform (CWT) is defined as the signal's integration multiplied by a scaled and shifted wavelet function (scale, position, time) taken over the signal's time span. The signal f L2 ()'s Continuous Wavelet Transform (CWT) is defined mathematically as.

For the recovery of signal f(t), the Inverse Wavelet Transform (IWT) is as follows:

A discretized variant of CWT is the wavelet expansion/series. Thus,

$$f \in L_2(\mathbb{R}) \Rightarrow f(x) = \sum_i \sum_j \langle f, \psi_{i,j} \rangle \psi_{i,j}$$

There are discrete versions of the Fourier and Wavelet transforms. This is obtained by substituting a finite sum for the integral. The Discrete Fourier Transform (DFT) is derived from the Fourier transform, and the Discrete Wavelet Transform is derived from the Wavelet Transform in the same way. DFT is usually computed using a Fast Fourier transform technique, because the computation time is lowered from a power of two to O. (nlogn). The wavelet coefficients form a matrix in the DWT. Wavelet filter coefficients are a set of numbers that describe a certain family of wavelets. Unlike DFT, we don't need a scaling function or a wavelet to compute DWT of a signal; instead, we only need a few simple digital filters.

## WAVELET PACKETS

Wavelet packets are a simple yet powerful extension of wavelets and MRA. It is commonly known that splitting Vj into Vj 1 and Wj 1 yields the standard MRA. The decomposition L2 () is obtained by repeating the same technique for Vj 1 recursively. If we utilize the "splitting trick" on the Wj spaces, we get the wavelet packets as the basis functions. Consider the following scenario: If V3 = V0 W0 W1 W2 is used three times, we get a wavelet packet with the following basis functions: 1992 (Beylkin).

Using the complete form, wavelet packet transform, increases the DWT's versatility and efficacy (WPT). WPT has the advantage of utilizing both low and high frequency components, referred to as approximation and details, respectively. Unlike DWT, which solely decomposes low-frequency components.Wavelet packet decomposition, in general, separates the frequency space into distinct pieces and provides for better signal frequency localization. Wavelet packet expansions have been applied in the solution of integral equations in recent years.

## MULTI DIMENSIONAL WAVELETS

Wavelets exist in higher dimensions as well. Two-dimensional signals, such as images, are one example. Tensor products are a simple approach to obtain higher-dimensional signals. Consider the following example of two-dimensional extension:

(x, y) = (x, y) = (x, y) = (x, y) = ( (x).defineV = f; f (x, y) = (x I y j), l2 ( 2) I j; f (x, y) = (x I y j); f (x, y) = (x I y j); f (x, y) = (x I y j); f (x, y) = (x I y j); If (x k)k is an orthonormal set, the orthonormal basis of V0 is (x I y j).

In two dimensions, wavelet decomposition is:

## WAVELET ANALYSIS- APPLICATION

The development of wavelets is inextricably linked to practical applications. The fact that wavelets' analytical properties are far more complicated makes them less useful for working mathematicians (but things are beginning to change). Signal analysis and image processing are the two applications where wavelets have had the most success. Purification, filtering (de-noising), efficient storage, retrieval, and transmission of time-signal and picture data, as well as their compression, are all included in the word processing. Wavelets can be successfully employed in many fundamental areas other than mathematics and physics, such as medicine, geophysics, astrophysics, social sciences, nano-science, wireless communication, and so on, because to their coherent features.

## WAVELETS IN NUMERICAL ANALYSIS

From the beginning, one of the reasons for the development of wavelet theory was the solution of differential and integral equations. Nowadays, wavelets are commonly employed in scientific computing and numerical analysis of differential, integral, and integro-differential equations. The numerical treatment of ordinary, delayed, and partial differential equations is one of the main fields where wavelets are gaining popularity. In Fluid Mechanics, wavelet-based numerical approaches for the resolution of evolutionary partial differential equations were examined. In order to compute eigenvalues and solve Regular Sturm-eigenvalue Liouville's problems (SLEP) with Dirichlet's boundary conditions, Haar wavelets were used. The combination of implicit Runge-Kutta method is currently the most effective approach for solving stiff differential equations encountered in Chemical engineering. To some extent, these strategies have not been successful in lowering calculating work and time. The Single Term Haar Wavelet Series (STHWS) technique, on the other hand, has proven to be more effective in solving systems with slightly to highly stiff differential equations. Wavelets are superior at solving partial differential equations in fluid dynamics, thanks to their hierarchical character, which has made them appealing to other domains as well. Several academics look at the suitability and applicability of wavelet approaches to problems that arise frequently in fluid dynamics. In the solution of linear and nonlinear integral equations with separable kernel, Haar wavelet packets have been used. Despite its capacity to solve partial differential equations quickly, the multigrid approach has downsides such as ease of implementation and rapid convergence. The wavelet-multigrid technique, which was recently proposed, offers an appealing alternative to this. The overall goal of the research is to illustrate the benefits of wavelet or wavelet-based approaches over standard numerical methods.

In retrospect, there is a major difference between the two formulations when it comes to numerical analysis. On the one hand, discretizing a differential equation produces a sparse matrix with a big condition number, while discretizing an integral equation produces a dense matrix with a small condition number. It can be seen that wavelet theory has traditionally been useful in the analysis of problems involving differential equations. In the 1990s, the wavelet approach was first used to solve differential and integral equations, and since then, hybridization of wavelet and classical methods has resulted in robust and stable numerical methods that do not sacrifice speed or accuracy.

Finally, any subject that has benefited greatly from wavelets is numerical analysis. The reason for this is that wavelet basis has qualities that are similar to finite element and spectral basis. It aids in the identification and localization of singularities, resulting in good approximation in smooth regions. The approximation to the solution is given by the oscillation property. In the late 1990s, numerical approaches based on wavelets for the resolution of partial differential equations, notably those designed in fluid mechanics, were researched. One difficulty that arises in this context is whether the Navier-Stokes (N-S) equation yields a non-smooth solution when smooth data is used. Is there anyone who can address this question using wavelets? The answer is yes, according to numerical evidence. However, a thorough mathematical understanding of the N-S equations-modeled events remains elusive.

## WAVELETS IN SOFT COMPUTING

The twenty-first and twenty-first centuries will be based entirely on computer systems. Soft computing is the process of solving mathematical problems with limited resources such as processing power and memory. In computational issues, the precision of the solution can be predetermined, which eliminates the option of using multiresolution analysis to achieve the accuracy sought. Wavelet analysis and multiresolution analysis are natural complements to soft computing methodologies. In general, multiresolution analysis is effective for determining space and defining precision.

## WAVELETS IN MEDICINE AND BIOLOGY

Because electrocardiography (ECG) indicates cardiac condition, it is critical to process and detect it correctly for better diagnosis and therapy. These signals are quasi-periodic, non-stationary, and fluctuate in performance with time. For improved clinical diagnosis, it should analyze data in both time and frequency. As a result, wavelet is the best option. The wavelet transform is also used in research and clinical diagnostics for electroencephalography (EEG) data, DNA analysis, protein analysis, blood pressure, heart rate, brain rhythms, and many other things. As a result, the wavelet transform becomes a useful mathematical/clinical tool for assessing different signal types.

## WAVELETS IN FLUID DYNAMICS

Wavelets, as a novel mathematical tool, provide new perspectives on present methodologies and will lead to a better understanding of turbulent flows. It aids in the separation of incoherent and coherent components in turbulent flows. Fourier analysis cannot supply this information because it is incapable of disentanglements. Fourier analysis simply delivers averages across time and does not provide any local data.

## WAVELETS IN OPTIMIZATION AND CONTROL THEORY

To achieve the best results, optimization is an essential aspect of every industry. There are several ways for computing optimal results and optimal control that can be found in textbooks and publications. Although these methods are complex, the computing element of optimum control becomes tough since it includes the numerical solution of a two-point boundary value problem. The solution of differential equations is a need of analysis for optimizing dynamic systems. Wavelets have recently been utilized to solve issues in calculus of variations, optimization theory, control theory, and other fields. Numerical solutions to SLEPs have been obtained using optimization theory and wavelets approaches, with a trade-off between speed and accuracy.

## WAVELETS IN COSMOLOGY AND ASTROPHYSICS

Another use of the wavelet transform is to investigate power law signals that can be found in a variety of astrophysical sources, such as detecting the intensity of light on solar surfaces. Wavelets have been employed in cosmology to analyze the spatial direction of galaxies. Because of the nature of signals and images, astronomical uses of wavelets differ from other applications. In the examination of astrophysical images, CWT, both in Euclidian and spherical forms, is a strong tool.

## WAVELETS IN CLIMATOLOGY

Wavelets have been effective in a variety of applications, but image processing is one of the most important. In satellite imaging, image compression is widely used. Denoising images is also taken into account when storing, transmitting, and retrieving satellite images. Wavelets have been effectively employed to assess air layer turbulence and ocean floor bathymetry or topography. Wavelet transformations are used to study temperature and wind fluctuations in order to provide quick and precise predictions.

## WAVELETS IN ELECTROMAGNETICS AND ACOUSTICS

Wavelets can be used to solve Maxwell's equation or to describe electromagnetic waves in general. Wavelet theory's basic operations, translation and dilation, are contained in a wide group of symmetries (the Conformal group C of space-time) in which Maxwell's equations in allowed space remain constant. Acoustic waves are the solution of wave equations in terms of space and time parameters. Conformal group C turns electromagnetic waves and acoustic wavelets into one another for this reason. As a result, creating acoustic wavelets and analyzing their scattering may be done in the same way as electromagnetic waves can.

## WAVELETS IN INDUSTRY AND COMMERCE

Electricity has always been a basic demand, but its reliance on industry, commerce, and services has resulted in the establishment of several restrictions aimed at improving power quality. The major goal is to reduce end-user device and/or process misbehaviors and losses. Statistical analysis of adaptive decomposition signals, describing and identifying instabilities in power systems using wavelet transform methods are among the ways presented. With the help of the Daubechies family, the Wavelet transform essentially offers automatic recognition of power quality disturbance waveforms. Wavelet algorithms of many forms have been investigated for detecting power quality issues.

Another field based on degrees of truth that is used in computers is fuzzy logic, which aids in the modeling of non-linear functions of arbitrary complexity. A fuzzy system can be developed with any set of input-output data. When fuzzy logic and wavelet theory are combined, spectral analysis becomes a powerful tool. Spectral analysis in the wavelet domain can be used to expand fuzzy logic to the frequency domain. As a result, although though they belong to different fields, fuzzy logic and wavelet theory are considered to complement each other. The following is an example of using fuzzy logic in conjunction with the wavelet transform: If the signal's high-frequency component is oversized, the wavelet coefficients are used to calculate the degree of membership. This collaboration opens the door to a slew of new possibilities in consumer durables manufacture.

## WAVELETS IN MANUFACTURING SCIENCE

Modern manufacturing makes heavy use of machine tools, which necessitates the ongoing reduction of unplanned machine downtime in order to provide cost-effective, dependable, and high-quality production. These machines should be advanced on a regular basis, based on cutting-edge science that can provide machine condition monitoring, fault diagnostics, and forecasting of remaining service life. The adaptive multi resolution capabilities of the wavelet transform has shown to be a useful and powerful mathematical tool in achieving these objectives.

## WAVELETS IN GEOSCIENCES

Wavelets were first effectively employed in geophysical research in 1984. Orthonormal wavelets, for example, were used in the research of air layer turbulence. Wavelet theory has been used in a variety of geophysical applications, including the interpretation of maritime seismic data and the characterisation of hydraulic conductivity distributions. Wavelet's ability to analyze and synthesize geo seismic signals in the research of air layer turbulence and corn crop turbulence has made it possible to develop tsunami warning systems in the long run.

## WAVELETS IN MULTIMEDIA

The wavelet transform is well recognized for its use in the processing of audio signals (speech/music) and the compression of still and video images. MP3, JPEG, and MPEG are Discrete Cosine Transform (DCT)-based computer algorithms (DCT). JPEG2000, based on DWT and the biorthogonal basis of Daubechies wavelets, is the current international standard for image compression. JPEG 2000, an image compression format, has a number of advantages over JPEG. The flexibility of the code stream is one of JPEG 2000's primary advantages. This means that after compression, the code stream can be shortened at any time. Aside from truncation, the code stream is scalable, which means it can be decoded in a variety of ways to produce a lower-resolution image. This eliminates the trade-off between image size and quality. The digital fingerprint image processing standard, approved by the FBI in 1993, is one of the most well-known wavelet-based applications. Wavelet transforms have also been used in communication systems. Wavelet OFDM, for example, is the basic modulation scheme utilized in Panasonic's HD-PLC power line communication technology.

Numerical methods can be used to solve some equations, such as the Fisher's equation and the Van-der-Pol equation, for which there is no direct solution. Numerous numerical techniques for solving differential and integral equations are available in the literature for a better approximation of the solution with less error. Low order approaches include the finite difference (FD), finite element (FE), and finite volume (FV) methods. Weighted residuals methods are also occasionally taken into consideration. WRMs (Weighted Residual Methods) (Finlayson, 1973) are based on the notion that a solution can be roughly or piecewise analytically analyzed. The technique makes an effort to reduce error. For instance, finite differences concentrate on reducing error at the selected grid points. Weighted residual methods are a special class of methods that define a certain technique by minimizing an integral mistake in a particular way. The approximation techniques that determine the differential equation's solution in the form of functions are the weighted residuals approaches.

Since the 1980s, the terms wavelet and ondelet have drawn the attention of scientists. Due to its effective use in signal and image processing, wavelet analysis gained importance. The hierarchical translation and dilation of a single function produced a smooth orthonormal foundation that was highly helpful in creating signal compression methods. The wavelet theory was significant from the perspectives of harmonic analysis and approximation theory. Wavelet theory and applications have seen tremendous growth and attention due to the positive characteristics of wavelets. A relatively new and developing field of mathematical study is wavelet theory. Wavelets have recently been used in a variety of fields, including statistics, engineering, biology, physics, chemistry, and even time series analysis. Wavelets are regarded as an effective tool in the fields of approximation theory, image processing, and matrix theory. Initial and boundary value problems have been numerically solved using various wavelet types and approximation algorithms.

## CONCLUSION

Wavelets are now being used to solve differential, integral, and integro-differential problems numerically. Numerous techniques have been researched from both a theoretical and computational perspective. Wavelets have a number of intriguing uses. Over the past few years, many wavelets-related applications have been created, ranging from data fusion to microarray analysis to biological imaging.

A wavelet is an oscillation that resembles a wave and has an amplitude that begins at zero, rises, and then falls back to zero. Typically, wavelets are created with the intention of having particular characteristics that make them helpful for signal processing. In order to extract information from an unknown signal, wavelets can be merged with sections of the signal using the reverse, shift, multiply, and sum procedure known as convolution. Wavelets are a mathematical tool that may be used to extract information from a variety of data types, including but not limited to audio signals and visual data. Data will be deconstructed using a set of complimentary wavelets in a way that leaves no gaps or overlaps and is mathematically reversible. In wavelet-based compression/decompression techniques where it is desired to retrieve the original information with low loss, sets of complimentary wavelets are therefore helpful. What is left is a smoother representation of the original signal with its low frequency components unaltered if we are just interested in the low frequency portion and discard the high frequency portion. Alternately, if the high frequency portion is what we are most interested in, we might be able to ignore the low frequency portion. All wavelets take the same approach of splitting a signal into two halves. Different sizes or resolutions of data are processed by wavelet algorithms. If we saw a signal via a big "window," we would see unsightly details. Similar to this, we would see a little "window" if we observed a signal with one. Wavelets allow a wide range of operators and functions to be accurately represented. Consider a camera lens that enables both taking wide-angle landscape photos and zooming in on minute detail that is often invisible to the human eye as an analogy for how wavelets operate. Beginning with the concepts of Fourier theory, which represents functions in terms of a series of sine and cosine functions, is a common starting point for explaining how wavelets work (having infinite support).

A drawback of wavelet is that the transformation only represents the data at a limited number of resolution levels, with each level's representation having a frequency that is about twice that of the one before it. Wavelets are a family of functions made by the transformation and dilation of a single function known as the mother wavelet. The Haar wavelets, Daubechies wavelets, and Meyer's wavelets are just a few types of wavelets with various characteristics.

The Galerkin or collocation technique serves as the foundation for the wavelet techniques for solving differential equations. Utilizing the Haar wavelet family is one option for this kind of problem. As piecewise constant functions make up Haar wavelets, which are actually Daubechies of order 1, they are the most basic orthonormal wavelets with a compact support. The discontinuity of the Haar wavelets is a flaw. Direct application of the Haar wavelets for solving differential and integral problems is not viable since derivatives do not exist at the breaking points. There are a few options for escaping this predicament.

This serves as motivation for the research and use of numerical techniques for approximating integrals and derivatives. Thus, one of the practical methods for locating approximations to the differential, integral, and integro-differential equations is numerical approximation by Haar wavelets.

The current state of wavelet theory is defined by a healthy link between model and applications. Diverse applications that have utilized and affected wavelet theory have been proposed by mathematicians in partnership with various subject experts and scientists. Overview papers are very valuable in fast changing and evolving topics, and many of them are worthwhile that deal with wavelets and are currently available. This thesis is a first step in the right path, and it tries to shed light on continuing research. Overall, wavelets have piqued interest and sparked interest in theoretical and applied sciences, particularly in recent years. In reality, the fields that use wavelets are progressing at such a quick pace that the meaning of wavelet analysis is always changing. The true issue is to identify a field of science or technology where wavelet techniques haven't been used or at least attempted and tested

## REFERENCES


1) Schneider, K. & Vasilyev, O.V. (2010) Wavelet Methods in Computational Fluid Dynamics. Annual Review of Fluid Mechanics. 42(1). 473-503.
2) Sepulveda, F., Wells, D.M. & Vaughan, C.L. (1993) A neural network representation of electromyography and joint dynamics in human gait. Journal of Biomechanics. 26 (2). 101-109.
3) Shalchyan, V., Jensen, W. & Farina, D. (2012) Spike Detection and Clustering with Unsupervised Wavelet Optimization in Extracellular Neural Recording, IEEE Transactions on Biomedical Engineering, 59 (9). pp. 2576-2585.
4) Sher A., Tadros R. O., (2017) 'Endograft sizing for abdominal aortic aneurysm' in Kirali K. (ed), Aortic Aneurysm, Intechopen pp. 109-127.
5) Sherlock, B.G. & Monro, D.M. (1996) Optimized wavelets for fingerprint compression. IEEE International Conference on Acoustics, Speech, and Signal Processing, ICASSP, Atlanta, IEEE, pp. 1447-1450.
6) Sinha, D. & Tewfik, A.H. (1992) Synthesis/coding of audio signals using optimized wavelets. IEEE International Conference, Acoustics, Speech, and Signal Processing, ICASSP, IEEE vol. 1, pp. I113-116.
7) Sinha, D. & Tewfik, A.H. (1993) Low bit rate transparent audio compression using adapted wavelets. IEEE Transactions on Signal Processing. 41 (12). pp. 3463-3479.
8) Sonal, D.K. (2005) A study of various image compression techniques. PhD thesis. Guru Jhambheswar University of Science and Technology.
9) Starnes B. W., Mehta, M., and Veith, F. J., (eds) (2017) Ruptured Abdominal aortic aneurysm, Switzerland, Springer International Publishing AG.
10) Stein, P.D., M.D. & Sabbah, H.N. (1976) Turbulent Blood Flow in the Ascending Aorta of Humans with Normal and Diseased Aortic Valves. Circulation Research. 39 (1). 58- 65.

11) Sultan, M.S., Martins, N., Ferreira, M.J. & Coimbra, M.T. (2016) Segmentation of bones & MCP joint region of the hand from ultrasound images. Proceedings of IEEE 38th Annual Int. Conf. of Eng.in Medicine and Biology Society (EMBC). pp. 4097-4100.

12) Sundar, A., Das, C. & Pahwa, V. (2016) Denoising Knee Joint Vibration Signals Using Variational Mode Decomposition. Proceedings of 3rd international conference on Information Systems Design and Intelligent Applications. 433. pp. 719-729.

13) Taubman, D.S. & Marcellin, M.W. (2002) Image compression fundamentals, standards and practice. JPEG2000 Image Compression Fundamentals, Standards and Practice. The Kulwer academic publisher, pp. 3-21.

14) Tian, J. & Wells, R.O. (1998) Embedded image coding using wavelet difference reduction; Wavelet image and video compression. The International Series in Engineering and Computer Science. 289-301.

15) Torzilli, P.A. & Mow, V.C. (1976) On the fundamental fluid transport mechanics through normal pathological articular cartilage during function–II. Jl. of Biomech. 9(9), 587-606.

**16)** Tracy, A.R., (2005) Abdominal Aortic Aneurysm Resection Transperitoneal Approach. The Surgical Technologists, 15-23.